Horizonte infinito:

Cuando economía es de T periodos:

$$\sum_{t=1}^{T} \frac{c_t}{(1+r_1)\cdots(1+r_{t-1})} + \frac{b_T}{(1+r_1)\cdots(1+r_{T-1})} = \sum_{t=1}^{T} \frac{y_t}{(1+r_1)\cdots(1+r_{t-1})} + b_0(1+r_0)$$

$$\Leftrightarrow \sum_{t=1}^{T} P_t c_t + \frac{b_T}{(1+r_1)\cdots(1+r_{T-1})} = \sum_{t=1}^{T} P_t y_t + b_0(1+r_0)$$

$$\Rightarrow \sum_{t=1}^{\infty} P_t c_t + \lim_{t\to\infty} \frac{b_T}{(1+r_1)\cdots(1+r_{t-1})} = \sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0)$$

Queremos que $\underbrace{\sum_{t=1}^{\infty} P_t c_t}_{\text{valor presente del consumo}} \leq \underbrace{\sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0)}_{\text{valor presente de ingresos (riqueza.}}$

$$\Leftrightarrow \boxed{\lim_{T\to\infty} \frac{b_T}{(1+r_1)\cdots(1+r_{T-1})} \geq 0}$$ — condición de <u>No-ponzi</u>

Esta restricción previene que existan esquemas de deuda donde la deuda de hogares crezca al infinito.

OJO: esto <u>NO</u> significa que hogares no puedan tener deuda.

Ej: $b_1 = -1$, $b_2 = -1$, $b_3 = -1$, ...

si $r_t > 0$

$$\lim_{T\to\infty} \frac{b_T}{(1+r_1)\cdots(1+r_{T-1})} = 0$$ → esto previene que deuda crezca de manera explosiva.

2 versiones del problema:

$$\max_{c_1, c_2, \dots} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t) \quad \text{s.a.} \quad \sum_{t=1}^{\infty} p_t c_t = \sum_{t=1}^{\infty} p_t y_t + b_0(1+r_0)$$

Const. presup. intertemporal.

$$\max_{\substack{c_1, \dots \\ b_1, \dots}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t) \quad \text{s.a.} \quad c_t + b_t = y_t + (1+r_{t-1}) b_{t-1}$$

$$\lim_{T \to \infty} \frac{b_T}{(1+r_1) \cdots (1+r_{T-1})} \geq 0$$

versión 1:

$$\mathcal{L} = \sum_{t=1}^{\infty} \beta^{t-1} u(c_t) + \lambda \left(\sum_{t=1}^{\infty} p_t y_t + b_0(1+r_0) - \sum_{t=1}^{\infty} p_t c_t \right)$$

$$[c_t]: \quad \beta^{t-1} u'(c_t) - \lambda p_t = 0 \qquad \Rightarrow \quad \beta^{t-1} u'(c_t) = \lambda p_t$$

$$[\lambda]: \quad \sum_{t=1}^{\infty} p_t c_t = \sum_{t=1}^{\infty} p_t y_t + b_0(1+r_0) \qquad \beta^{t} u'(c_{t+1}) = \lambda p_{t+1}$$

$$\frac{\cancel{\beta^{t}} u'(c_{t+1})}{\cancel{\beta^{t-1}} u'(c_t)} = \frac{\cancel{\lambda} p_{t+1}}{\cancel{\lambda} p_t} \iff \beta u'(c_{t+1}) = \frac{p_{t+1}}{p_t} u'(c_t)$$

$$p_t = \frac{1}{(1+r_1) \cdots (1+r_{t-1})} \Rightarrow \frac{p_{t+1}}{p_t} = \frac{\dfrac{1}{\cancel{(1+r_1)} \cdots \cancel{(1+r_{t-1})}(1+r_t)}}{\dfrac{1}{\cancel{(1+r_1)} \cdots \cancel{(1+r_{t-1})}}} = \frac{1}{1+r_t}$$

$$\Rightarrow \beta u'(c_{t+1}) = \frac{1}{1+r_t} u'(c_t) \iff \boxed{u'(c_t) = \beta(1+r_t) u'(c_{t+1})}$$

$$\boxed{\sum_{t=1}^{\infty} p_t c_t = \sum_{t=1}^{\infty} p_t y_t + b_0(1+r_0)}$$

Versión 2:

$$\mathcal{L} = \sum_{t=1}^{\infty} \beta^{t-1} u(c_t) + \sum_{t=1}^{\infty} \lambda_t \left(y_t + (1+r_{t-1}) b_{t-1} - c_t - b_t \right)$$

$[c_t]$: $\beta^{t-1} u'(c_t) = \lambda_t$

$[b_t]$: $\lambda_t = \lambda_{t+1}(1+r_t)$

$[\lambda_t]$: $\boxed{y_t + (1+r_{t-1}) b_{t-1} = c_t + b_t}$ (**)

$\Big\} \; \cancel{\beta^{t-1}} u'(c_t) = \cancel{\beta^{t}} u'(c_{t+1})(1+r_t)$

$\Rightarrow \boxed{u'(c_t) = \beta(1+r_t) u'(c_{t+1})}$ (*)

Con infinitos periodos, estas condiciones son <u>necesarias</u> pero <u>NO</u> suficientes para un óptimo.

Ej: Supongamos que $y_t = 1$, $r_t = \rho$, $b_0 = 0$

$$u'(c_t) = \beta(1+r_t) u'(c_{t+1}) \qquad \beta = \frac{1}{1+\rho}$$

$$(1+r_t)\beta = 1+\rho \cdot \frac{1}{1+\rho} = 1$$

$$\Rightarrow u'(c_{t+1}) = u'(c_t) \iff \boxed{c_t = c_{t+1}}$$ — hogar consume cantidad constante

Solución óptima $\boxed{c_t^* = 1}$

Tomemos $c_t = 1 - \varepsilon$ $\forall t$. $\rightarrow$ consumo es subóptimo.

Como $c_t = 1 - \varepsilon$ es constante $\Rightarrow$ Euler se satisface ✓

$b_1 = y_1 - c_1 = 1 - (1-\varepsilon) = \varepsilon$ $\longrightarrow$ c_t y b_t cumplen (*) y (**)

$b_2 = y_2 + (1+r_1) b_1 - c_1 = 1 + (1+\rho)\varepsilon - (1-\varepsilon) = \cancel{1} + \varepsilon + \rho\varepsilon - \cancel{1} + \varepsilon$
$= \varepsilon + (1+\rho)\varepsilon$

$$\frac{b_2}{1+r_1} = \frac{\varepsilon}{1+\rho} + \varepsilon$$

$$\frac{b_3}{(1+r_1)(1+r_2)} = \varepsilon + \frac{\varepsilon}{1+\rho} + \frac{\varepsilon}{(1+\rho)^2}$$

$b_1 = \varepsilon$
$b_2 = \varepsilon + (1+\rho)\varepsilon$
$b_3 = \varepsilon + (1+\rho)\varepsilon + (1+\rho)^2\varepsilon$
$\vdots$

$$\vdots$$

$$\frac{b_T}{(1+r_1)\cdots(1+r_{T-1})} = \sum_{t=1}^{T} \varepsilon \left(\frac{1}{1+\rho}\right)^{t-1}$$

$$\lim_{T\to\infty} \frac{b_T}{(1+r_1)\cdots(1+r_{T-1})} = \lim_{T\to\infty} \sum_{t=1}^{\infty} \varepsilon \left(\frac{1}{1+\rho}\right)^{t-1}$$

$$= \varepsilon \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho}\right)^{t-1} = \frac{\varepsilon(1+\rho)}{\rho} > 0$$

Debemos agregar: $\underbrace{\lim_{T\to\infty} \frac{b_T}{(1+r_1)\cdots(1+r_{T-1})} = 0}_{\text{condición de transversalidad.}}$

Para tener un conjunto de condiciones necesarias + suficientes.
Es decir, solución al problema del hogar.

$$\boxed{\begin{aligned} u'(c_t) &= \beta(1+r_t)\,u'(c_{t+1}) \\ c_t + b_t &= y_t + (1+r_{t-1})\,b_{t-1} \\ \lim_{T\to\infty} \frac{b_T}{(1+r_1)\cdots(1+r_{T-1})} &= 0 \end{aligned}}$$

Ej: $u(c_t) = \ln(c_t)$

$$\mathcal{L} = \sum_{t=1}^{\infty} \ln c_t + \lambda\left(\sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0) - \sum_{t=1}^{\infty} P_t c_t\right)$$

$$\Leftrightarrow \quad \boxed{c_{t+1} = \beta \frac{P_t}{P_{t+1}} c_t = \beta(1+r_t)\,c_t}$$

$$\boxed{\sum_{t=1}^{\infty} P_t c_t = \sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0)}$$

(*)

$P_{t+1} c_{t+1} = \beta P_t c_t$
$P_t c_t = \beta P_{t-1} c_{t-1}$
$\vdots$

$P_t c_t = \beta P_{t-1} c_{t-1}$
$P_{t-1} c_{t-1} = \beta P_{t-2} c_{t-2}$
$P_{t-2} c_{t-2} = \beta P_{t-3} c_{t-3}$
$\vdots \quad \vdots$

$$P_t C_t = \beta\left(\beta P_{t-2} C_{t-2}\right) = \beta\left(\beta\left(P_{t-3} C_{t-3}\right)\right) = \dots = \beta^{t-1} \cancel{P_1} C_1$$

$$\Rightarrow P_t C_t = \beta^{t-1} C_1$$

$$\sum_{t=1}^{\infty} P_t C_t = \sum_{t=1}^{\infty} \beta^{t-1} C_1 = C_1 \underbrace{\sum_{t=1}^{\infty} \beta^{t-1}} = \sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0)$$

$$\sum_{t=1}^{\infty} a^{t-1} = \frac{1}{1-a} \quad \text{si} \quad |a| < 1$$

$$\Rightarrow C_1 \left(\frac{1}{1-\beta}\right) = \sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0)$$